Chapter 5: Decision Tree Learning

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Fall 2022

Introducing Decision Tree Learning

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- Decision Tree Learning algorithms such as ID3 or CART use a measure of impurity (such as *entropy* or *Gini impurity*) and at each node, try to find a feature-condition pair that reduces impurity most.
- Such algorithms try to find a tree whose leaves S_v are pure, i.e. contain only one class.

Example of a Decision Tree

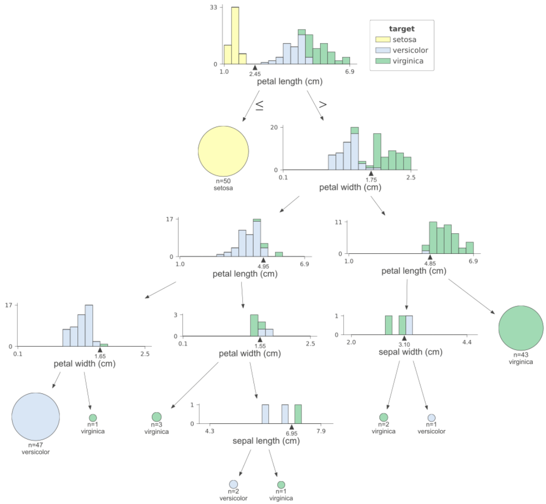

Figure: A decision tree for the iris dataset depicted using `dtreeviz` library.

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- In case we have a set S containing elements belonging to m different classes, and $p(k)$ is the probability of belonging to class k then $H(p)$ is a measure of "mixedness" (or impurity) of S in terms of classes.

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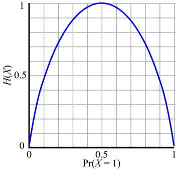

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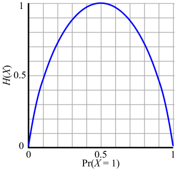

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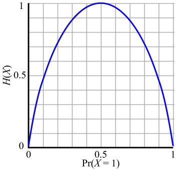

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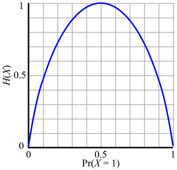

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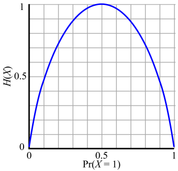

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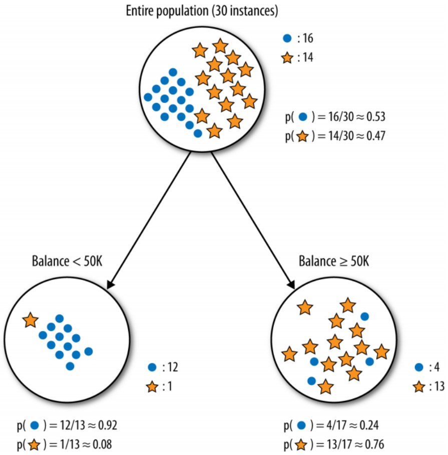

Figure: Compute the information gain for this splitting. Credit: Foster provost and Tom Fawcett

Another example of Information Gain

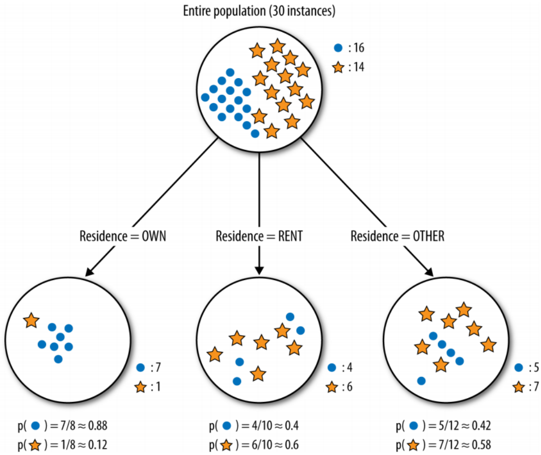

Figure: Another splitting of the same set. Credit: Foster Provost and Tom Fawcett

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- This way, starting from the root, the algorithm grows the decision tree. A node is taken as a leaf if its set is pure or if the set cannot be split by features anymore, or if a preset maximum depth is reached.

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- However finding the optimal tree w.r.t. L is NP-Complete and thus we use greedy algorithms to find a near-optimum.

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- In Scikit-Learn, decision tree classifier is provided by the class `tree.DecisionTreeClassifier` and uses the CART algorithm and currently does not support categorical features.

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  - `min_samples_split`: minimum number of samples a node must have to be allowed to split.

Overfitting and regularization in Decision Tree Learning (DTL) cont.

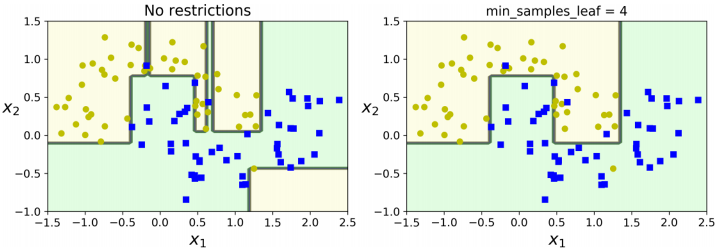

Figure: An example of Decision Tree Learning with and without regularization. Credit: Aurelien Geron

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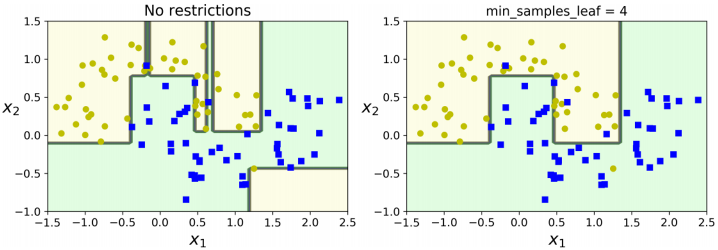

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- In Scikit, decision boundaries for a classifier can be drawn using the `inspection.DecisionBoundaryDisplay.from_estimator` function.

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- In Scikit, decision tree regression is provided by `tree.DecisionTreeRegressor`.

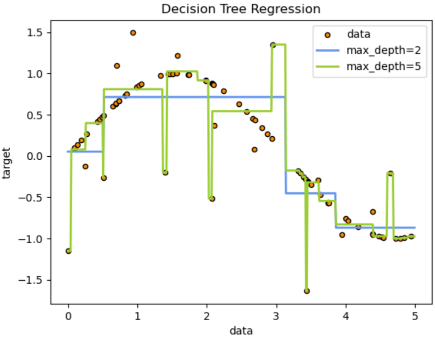

Figure: An example of Decision Tree Regression on sinusoidal data. We have only one feature which is plotted on the x-axis.

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