Chapter 3: Introduction to Classification

Reza Rezazadegan

Sharif University of Technology

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- This is the subject of **Logistic Regression** which is actually a classification method!

Logistic Regression

- In Logistic Regression we model our classifier by a function of the form

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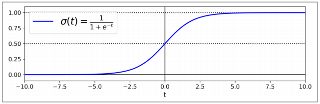

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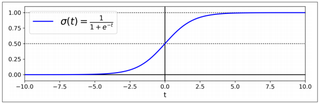

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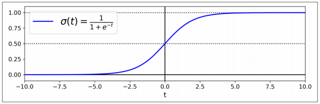

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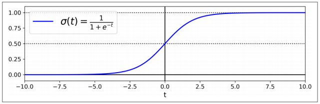

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- Later we see *generative models* which directly learn the joint probability distribution $p(\mathbf{x}, y)$.

Training Logistic Regression

- The loss function for Logistic Regression is given by

$$L(\mathbf{a}) = -\sum_{i=1}^{n} \log p(y = y_i | \mathbf{x}_i) = -\sum_{y_i=1} \log \sigma(\mathbf{x}) - \sum_{y_i=0} \log(1 - \sigma(\mathbf{x})) \tag{3}$$

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$$\nabla_{\mathbf{a}} L(\mathbf{a}) = \sum_{i=1}^{n} (y_i - \sigma(\mathbf{x}_i))\mathbf{x}_i \tag{5}$$

- There is no know closed-form solution for $\nabla_{\mathbf{a}} L(\mathbf{a}) = 0$.

The importance of paying attention to model hypotheses and confidence levels

- Not paying attention to the hypothesis of the model we use, and its confidence in the prediction, can have consequences.

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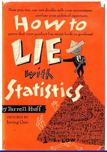

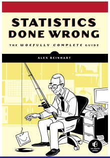

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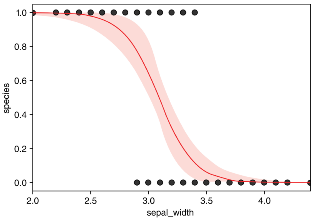

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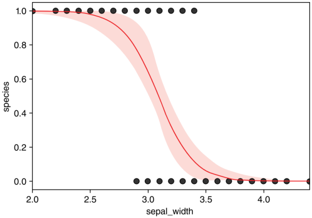

- Exercise: show that the *decision boundary* $p(y=1|\mathbf{x}) = p(y=0|\mathbf{x})$ is given by the hypersurface $\mathbf{a}^t\mathbf{x} = a_0 + a_1x_1 + a_2x_2 + \cdots + a_dx_d = 0$.

Measuring the performance of a binary classifier

- Confusion Matrix:
  Positive: belonging to the class 1, e.g. "sick".
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  True: classified correctly by our model.
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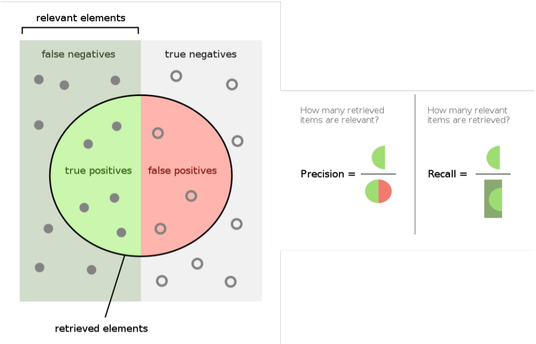

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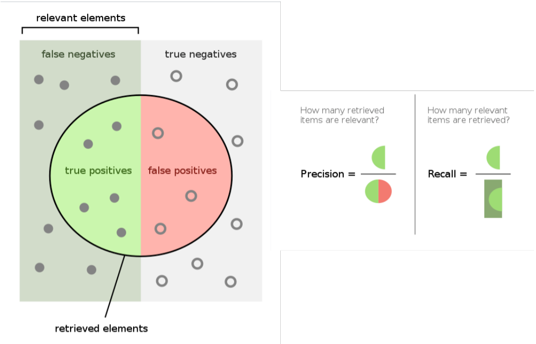

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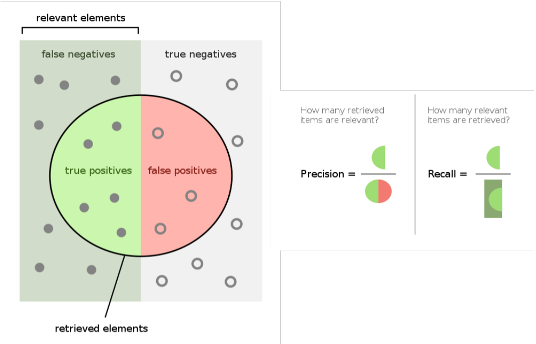

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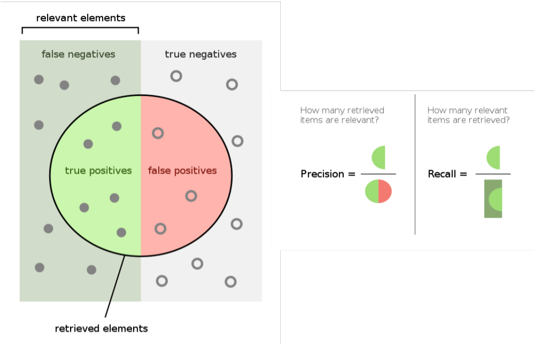

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- Accuracy is not a good measure if the two classes are imbalanced.

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- Recall=1 means no false negatives.

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- Precision=1 means no false positives.
- Recall=1 means no false negatives.
- Recall is important in medical or security applications of ML, and also in content filtering where false negatives can be costly.

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- Threshold too high: few positive predictions which are mostly correct; which means high precision but low recall!
- Threshold too low: a lot of positive predictions which contain a lot of false predictions; which means low precision but high recall!
- Stated differently: Recall measures how many of the positives are actually retrieved!

Precision-Recall Trade-off cont.

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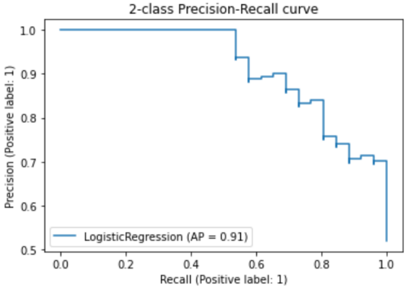

Figure: Precision-Recall curve for two species in the Iris dataset.

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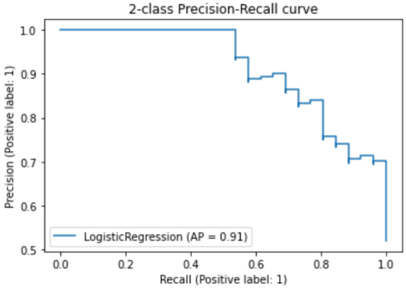

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The higher the area under the Precision-Recall curve the better the classifier.

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- Multi-class Logistic Regression is used when the classes are mutually exclusive.

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- **Cross Entropy** of two distributions q, p equals $-E_q[\log p]$ which means we have used a "wrong" distribution p to sample our data.

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- Minimizing the Cross Entropy loss function is done using the method of Gradient Descent.

- Logistic Regression for the 3 classes in the Iris dataset, using two features:

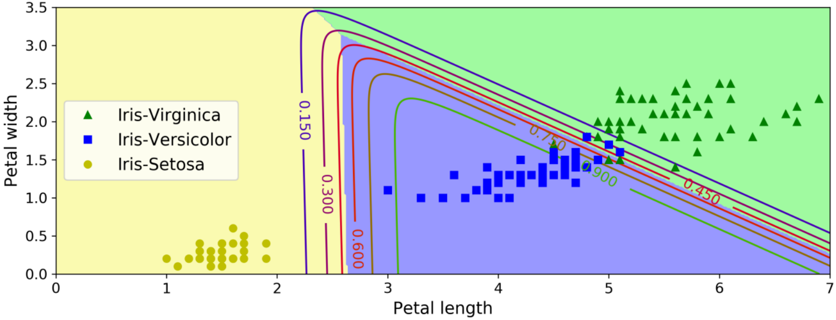

Figure: Credit: A. Geron, Hands-on Machine Learning

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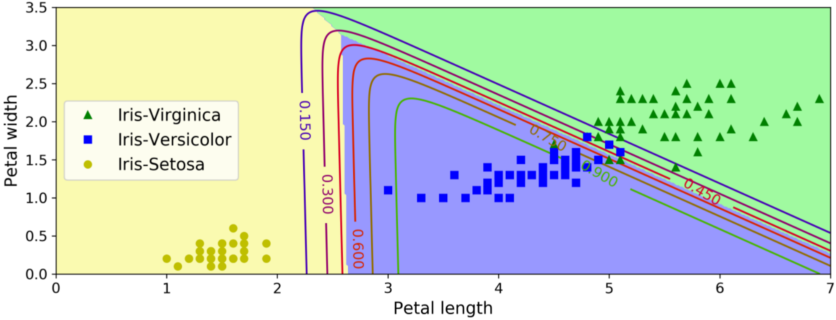

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- Exercise: show that the decision boundaries $p(y = k|\mathbf{x}) = p(y = l|\mathbf{x}) = 0.5$ between any two classes in Multi-class Logistic Regression are line segments.

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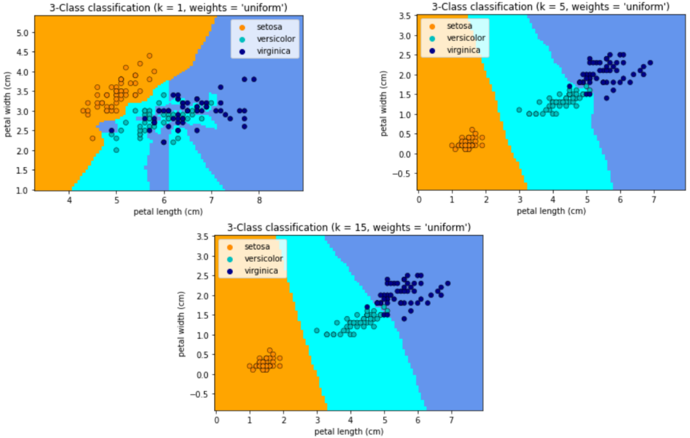
- In weighted KNN, the "vote" of each nearest neighbor is weighted by e.g. the inverse of its distance.
- KNN can be used as a Regression method too: $f(\mathbf{x}) = \sum_{i=1}^k f(\mathbf{x}_{n_i})$ where the $\mathbf{x}_{n_i}$ are the nearest neighbors of $\mathbf{x}$.


3-Class classification (k = 1, weights = 'uniform')
3-Class classification (k = 5, weights = 'uniform')
3-Class classification (k = 15, weights = 'uniform')
setosa
versicolor
virginica
petal width (cm)
petal length (cm)

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